IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Johnson-Nyquist noise for a 2D electron gas in a narrow channel

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1993 J. Phys.: Condens. Matter 5 L469

(http://iopscience.iop.org/0953-8984/5/39/003)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 171.66.16.96
The article was downloaded on 11/05/2010 at 01:52

Please note that terms and conditions apply.

J. Phys.: Condens. Matter 5 (1993) L469–L474. Printed in the UK

LETTER TO THE EDITOR

Johnson–Nyquist noise for a 2D electron gas in a narrow channel

O M Bulashenko and V A Kochelap

Institute of Semiconductor Physics, Academy of Sciences of the Ukraine, Prospekt Nauki 45, Kiev 252650, Ukraine

Received 26 May 1993

Abstract. The equilibrium current fluctuations (thermal noise) for a 2D degenerate electron gas bounded in a narrow channel have been calculated within the semiclassical Boltzmann-equation approach. The analytical formula for the autocorrelation function has been derived for the completely degenerate case. The associated noise spectrum has a non-Lorentzian shape with decreased zero-frequency plateau and smeared geometrical resonances at high frequencies caused by the restriction on electron motion in the transverse direction (classical size effect). Measurements of the size-dependent noise spectrum would give additional information about the edge scattering of electrons.

The noise properties of the electron gas in small-size conductors have attracted considerable interest during recent years [1–5]. The transition from diffusive to ballistic transport was found to be accompanied by new interesting phenomena: (i) the suppression of shot noise [1, 2]; (ii) the noise redistribution towards higher frequencies ('blue shift'), depending on the geometrical size of the sample [3–5]; (iii) the geometrical resonances in the spectrum [5], and so on.

In this letter the Johnson–Nyquist noise characteristics for a two-dimensional (2D) electron gas *bounded in a narrow channel* are presented. Within the semiclassical approach the autocorrelation function of the current fluctuations is calculated both analytically and by use of the Monte Carlo technique. The spatial correlation of the fluctuations is taken into account, which is essential in small-size conductors [6].

Consider a 2D electron gas in the xy plane laterally restricted by the diffusely reflected boundaries at $y = 0$ and $y = d$. The channel width d is assumed to be much wider than the Fermi wavelength. The electrons are scattered both in the bulk and at the boundaries. The length L in the x direction is much greater than the electron mean free path λ and terminated by contacts for measurement of the equilibrium current fluctuations.

It is known that the contacts can disturb the current flow in the near-contact regions. For low-dimensional systems this point was studied, e.g. in [7]. In our model those regions are of the order of the channel width d and we suppose $L \gg d$; hence, the contribution of the contacts to the noise phenomena can be neglected. Thus, the electron transport is characterized by the parameter $\gamma = \lambda/d$, which is the 'degree of ballistic transport' [3–5]. Varying γ from $\gamma \ll 1$ to $\gamma \gg 1$ we are going from entirely bulk scattering (diffusive regime) to entirely boundary scattering of electrons (ballistic or Knudsen regime of electron transport [4]).

0953-8984/93/390469+06$07.50 © 1993 IOP Publishing Ltd

The instantaneous short-circuit current $I(t)$ through the sample of length L can be expressed as a sum of the instant velocities of all carriers presented at the time t in the sample [8]

$$I(t) = \frac{e}{L}\sum_{i=1}^{N} v_{xi} = \frac{e}{L}\int \mathrm{d}\boldsymbol{r}\int \frac{2}{(2\pi)^2}\,\mathrm{d}\boldsymbol{k}\; v_x f(\boldsymbol{r},\boldsymbol{k},t). \tag{1}$$

Here e is the electronic charge, $\boldsymbol{r} \equiv (x, y)$ is the radius vector, $\boldsymbol{k} \equiv (k_x, k_y)$ is the wavevector, $v_x = \hbar k_x/m$ is the electron velocity component and m is the effective mass. The integration over $\boldsymbol{r}$ is taken over the channel area $L \times d$. $f(\boldsymbol{r},\boldsymbol{k},t)$ represents the electron distribution function (occupation numbers).

From this formula the average current $\langle I(t)\rangle$ under the equilibrium conditions is zero, because for this case $\langle f(\boldsymbol{r},\boldsymbol{k},t)\rangle = f_0(\boldsymbol{k})$, the Fermi–Dirac distribution function. But the instantaneous current $I(t)$ due to fluctuation of $f(\boldsymbol{r},\boldsymbol{k},t)$ is not zero and the current autocorrelation function is

$$C_I(t) = \langle \delta I(0), \delta I(t)\rangle$$
$$= \frac{e^2}{L^2}\frac{1}{(2\pi^2)^2}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}\boldsymbol{r}'\int \mathrm{d}\boldsymbol{k}\int \mathrm{d}\boldsymbol{k}'\; v_x v_x' \langle \delta f(\boldsymbol{r},\boldsymbol{k},t)\;\delta f(\boldsymbol{r}',\boldsymbol{k}',0)\rangle \tag{2}$$

where $\delta I(t) \equiv I(t) - \langle I(t)\rangle = I(t)$, $\delta f(\boldsymbol{r},\boldsymbol{k},t) = f(\boldsymbol{r},\boldsymbol{k},t) - f_0(\boldsymbol{k})$. In equation (2) the angle brackets indicate averaging over the initial time moment $t = 0$ (for a fixed value of t).

The correlation function $\langle \delta f(\boldsymbol{r},\boldsymbol{k},t)\;\delta f(\boldsymbol{r}',\boldsymbol{k}',0)\rangle$ satisfies for $t > 0$ the Boltzmann kinetic equation in the first set of variables [2, 9]. By neglecting interaction between electrons and assuming the scattering to be elastic one can write

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial \boldsymbol{r}} + \frac{1}{\tau}\right)\langle \delta f(\boldsymbol{r},\boldsymbol{k},t)\delta f(\boldsymbol{r}',\boldsymbol{k}',0)\rangle = 0. \tag{3}$$

The term $1/\tau$ corresponds to the collision integral in the relaxation-time approximation, with τ being the average time between collisions in the bulk.

Let us introduce the notation of the correlation function averaged over the longest dimension

$$\mathcal{K}(y,\boldsymbol{k},t;\,y',\boldsymbol{k}') = \frac{1}{L}\int_0^L \mathrm{d}x\int_0^L \mathrm{d}x'\;\langle \delta f(\boldsymbol{r},\boldsymbol{k},t)\;\delta f(\boldsymbol{r}',\boldsymbol{k}',0)\rangle. \tag{4}$$

After integration over x, x', the Boltzmann equation for this function will be

$$\left(\frac{\partial}{\partial t} + v_y\frac{\partial}{\partial y} + \frac{1}{\tau}\right)\mathcal{K}(y,\boldsymbol{k},t;\,y',\boldsymbol{k}') = 0. \tag{5}$$

The term with $v_x(\partial/\partial x)$ vanishes, being of the order of the small parameter λ/L. Using the expression for the equal-time correlation of the distribution function [9], one obtains the initial condition for (5) in the form

$$\mathcal{K}(y,\boldsymbol{k},0;\,y',\boldsymbol{k}') = 2\pi^2\,\delta(y-y')\,\delta(\boldsymbol{k}-\boldsymbol{k}')\;f_0(\boldsymbol{k})\;(1-f_0(\boldsymbol{k})) + \mathcal{K}_1(y,\boldsymbol{k},y',\boldsymbol{k}'). \tag{6}$$

The factor $(1 - f_0)$ in the first term of the RHS arise from the Fermi statistics under investigation. $\mathcal{K}_1$ is the off-diagonal term with respect to $\boldsymbol{k}, \boldsymbol{k}'$, giving a contribution for

the system with a fixed number of particles [9]. Being symmetric in our problem, it does not contribute to the noise, since integration of (2) over k_x, k_x' gives zero. Thus, the last term is not considered here, but in the non-equilibrium case it may be essential.

We consider the case of fully diffuse electron scattering at the boundaries, which destroys any correlation between the electron flow towards the boundary and the backward electron flow during the surface collision time τ_b. So, the boundary conditions can be given by

$$\mathcal{K}(0, \boldsymbol{k}, t; y', \boldsymbol{k}') = 0 \quad \text{for} \quad v_y > 0 \tag{7}$$

$$\mathcal{K}(d, \boldsymbol{k}, t; y', \boldsymbol{k}') = 0 \quad \text{for} \quad v_y < 0 \tag{8}$$

for $t \geqslant \tau_b$. Since $\tau_b \ll \tau, d/v_F$, the characteristic times of the problem, we can refer the boundary conditions (7) and (8) to the time $t = 0$, the initial condition (6) remaining valid everywhere except the edges ($y \neq 0; d$). Here, v_F is the Fermi velocity.

The solution of (5) for the function $\mathcal{K}$ under conditions (6)–(8) can be expressed by

$$\mathcal{K}(y, \boldsymbol{k}, t; y', \boldsymbol{k}') = 2\pi^2 e^{-t/\tau} \delta(y - y' - v_y t)\, \delta(\boldsymbol{k} - \boldsymbol{k}')\, f_0(\boldsymbol{k})\, [1 - f_0(\boldsymbol{k})]. \tag{9}$$

The function $\delta(y - y' - v_y t)$ represents the spatial correlation of the fluctuation. Substituting (4) and (9) into (2) and carrying out the integration over $\boldsymbol{k}'$, y and y', $C_I(t)$ becomes

$$C_I(t) = \frac{2e^2}{\pi^2} \frac{d}{L} e^{-t/\tau} \int_0^{md/\hbar t} dk_y \int_0^{\infty} dk_x\, v_x^2 \left(1 - \frac{v_y t}{d}\right) f_0(\boldsymbol{k})[1 - f_0(\boldsymbol{k})]. \tag{10}$$

For the completely degenerate case, where $f_0(1 - f_0) = k_B T\, \delta(\epsilon - \epsilon_F)$, the current autocorrelation function (10) can be evaluated analytically

$$C_I(t) = \frac{e^2 n_2}{m} \frac{d}{L} k_B T\, e^{-t/\tau}\, f(u) \qquad \text{where } u = \frac{\gamma t}{\tau} = \frac{v_F t}{d} \tag{11}$$

$$f(u) = \begin{cases} 1 - \dfrac{4}{3\pi} u & 0 < u < 1 \\ \dfrac{2}{\pi}(\arcsin u^{-1} + \frac{1}{3}(1 - u^{-2})^{1/2}(2u + u^{-1}) - \frac{2}{3}u) & u > 1 \end{cases}$$

$n_2 = k_F^2/2\pi$ is the 2D electron concentration, k_B is the Boltzmann constant, T is the temperature, and ϵ_F and k_F are the Fermi energy and momentum. The parameter u is the time in units of the transit time of electrons between the boundaries.

In the previous work [4] the Monte Carlo (MC) technique for calculating the velocity fluctuations in thin 3D metal films with diffuse surfaces has been proposed. In the present work on the basis of the same algorithm as in [4] the random motion of a single electron with the Fermi velocity (completely degenerate case) in the 2D channel has been simulated. The only distinction was in generating the random scattering angles for 2D geometry.

The velocity autocorrelation function $C_v(t)$ calculated by use of the MC method is compared in figure 1 with the current autocorrelation function given by equation (11). For a given number of collisions ($\sim 10^5$) the agreement is quite good down to $C(t) \sim 0.01\, C(0)$, indicating that the MC procedure is correct and it can be fruitfully applied to obtain the thermal noise characteristics for the degenerate case. However, in order to get $C_I(t)$ from $C_v(t)$, estimated by the MC technique for a single electron with the Fermi velocity, one must multiply the result by the factor $k_B T/\epsilon_F$ (in addition to the dimension term $ne^2 d/L$). The

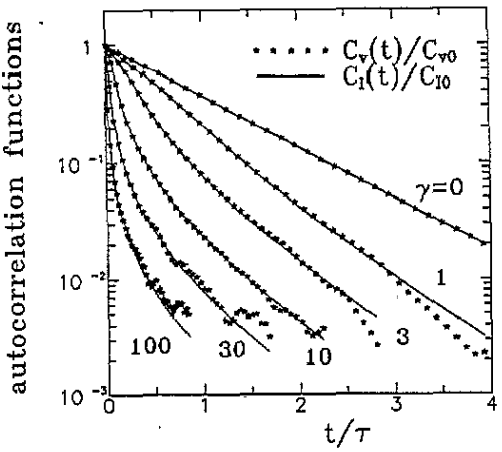

Figure 1. Velocity autocorrelation functions calculated by use of the Monte Carlo method (stars) and current autocorrelation functions obtained from analytical expression (11) (full curves), both in the degenerate limit for different values of γ. The normalization constants are $C_{v0} = \frac{1}{2}v_F^2$ and $C_{I0} = e^2 n_2 (k_B T/m)(d/L)$.

physical meaning of this factor is the fraction of electrons near the Fermi surface taking part in the fluctuations.

For a 3D thin film $C_I(t)$ can be also evaluated analytically by the same manner as equation (11) for the 2D case:

$$C_I(t) = \frac{e^2 n_3}{m}\frac{A}{L} k_B T \, e^{-t/\tau} \, f(u) \tag{12}$$

with

$$f(u) = \begin{cases} 1 - \frac{3}{8}u & 0 < u < 1 \\ \frac{3}{4}u^{-1}(1 - \frac{1}{6}u^{-2}) & u > 1 \end{cases}$$

where $n_3 = k_F^3/3\pi^2$ is the 3D electron concentration, A is the cross sectional area of the sample and the other notations are as previously defined in (11). This is the formula for the current autocorrelation function within the Fuchs size-effect model [4, 11] for fully diffuse electron scattering at the boundaries. $C_v(t)$ calculated from the MC procedure [4] can be related to the Johnson–Nyquist noise by multiplying the results by the factor $\frac{3}{2}k_B T/\epsilon_F$ (in addition to the dimension term ne^2A/L) in a similar manner to the 2D case mentioned above.

Let us return to the results for the 2D channel. The noise spectral density $S_I(\omega)$ was calculated as a Fourier transform of the autocorrelation function $C_I(t)$ (see figure 2). When $\gamma \to 0$, $u \to 0$ and $f(u) \to 1$, we have the ordinary Johnson–Nyquist noise with the Lorentzian spectrum. With increasing γ the low-frequency noise is suppressed with a redistribution toward higher frequencies and a remarkable deviation from the Lorentzian shape (the corresponding Lorentzian curves are shown by broken curves). One can also observe the smeared oscillations at frequencies corresponding to the time of flight between the boundaries d/v_F. The inset of figure 2 illustrates this statement: extrema of $S_I(\omega)$ normalized to the corresponding Lorentzians hold at the same values of $\omega d/v_F$ for different

scales of the channels. The origin of the oscillations is caused by the restriction on the electron motion in the y direction (the classical size effect), and their nature is similar to those in [5], where size effect occurs between the contacts of the 1D wire. Unlike the 1D case [5], where geometrical oscillations are more pronounced, for the 2D channel they are smeared.

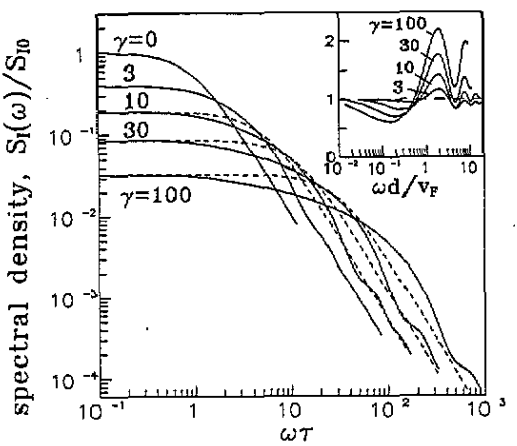

Figure 2. Spectral density of current fluctuations in the degenerate limit for different γ (full curves). The normalization constant is $S_{I0} = 4k_BT(n_2e^2\tau/m)(d/L)$. Inset: $S_I(\omega)$ normalized to the corresponding Lorentzians.

Due to the Nyquist theorem $S_I(0) = 4k_BTG$, with G being the conductance. Hence, the conductivity σ can be obtained from the current autocorrelation function by

$$\sigma = (L/d)\int_0^\infty C_I(t)\,\mathrm{d}t/k_BT.$$

For 2D and 3D cases, integrating equations (11) and (12) over the time t, and denoting $u = 1/s$, one gets

$$\sigma_{2D} = \frac{e^2n_2\tau}{m}\left((1-\frac{4}{\pi}\gamma\int_0^1 \mathrm{d}s\; s(1-s^2)^{1/2}(1-\mathrm{e}^{-1/\gamma s})\right) \tag{13}$$

$$\sigma_{3D} = \frac{e^2n_3\tau}{m}\left(1-\frac{3}{2}\gamma\int_0^1 \mathrm{d}s\; s(1-s^2)(1-\mathrm{e}^{-1/\gamma s})\right). \tag{14}$$

The second equation is the Fuchs formula for the conductivity in thin metal films [10]. Analogously the diffusion coefficients take the form (13), (14) and the corresponding equations coincide with those in [11]. It should be noted that the frequency-dependent $\sigma(\omega)$ can be obtained from (11) and (12) by the same manner.

In conclusion, the autocorrelation function for the thermal (Johnson–Nyquist) noise and the corresponding noise spectrum have been calculated for a 2D degenerate electron gas in a narrow channel with diffusely reflected boundaries. By diminishing the channel width the noise is redistributed toward higher frequencies with a suppression of its low-frequency magnitude. The noise spectrum has a non-Lorentzian shape with damped geometrical

resonances at frequencies corresponding to the time of flight d/v_F between the boundaries. The Monte Carlo approach discussed in the present and the previous papers [4] may be usefully applied for calculating both the conductivity and the thermal noise for the electron gas in the region with more complicated and partially reflected boundaries. The results for the other geometries of the low-dimensional channels will be published elsewhere [12].

To verify the theoretical conclusions of this work the following experiment on semiconductor heterostructures may be proposed. A narrow channel is formed in the plane of a 2D electron gas by a rough boundary at one edge and a specular reflected boundary, displaced by a voltage-operated gate, at the other. Varying the gate voltage, one can measure the low-frequency current noise versus channel width. As we calculated, the noise figure and the conductance of such channels are identical to those of the channel with diffuse boundaries on both sides but of twice the width. Electron scattering at the boundaries of narrow channels was shown to be diffuse, when it is produced by different techniques (for example, by focused ion beam, chemical etching, etc [13]). The measurements of the equilibrium size-dependent noise, proposed here, may be used as an additional method to study edge scattering for 2D electron gas channels.

This work is supported by a grant from the Ukrainian Council on Progress in Science and Engineering. We thank N A Zakhleniuk for discussion on the expression (6) for the equal-time correlation function.

References

[1] Lesovik G B 1989 *JETP Lett.* **49** 592
Büttiker M 1990 *Phys. Rev. Lett.* **65** 2901
Martin Th and Landauer R 1992 *Phys. Rev.* B **45** 1742
Büttiker M 1992 *Phys. Rev.* B **45** 3807
Beenakker C W J and Büttiker M 1992 *Phys. Rev.* B **46** 1889
[2] Beenakker C W J and van Houten H 1991 *Phys. Rev.* B **43** 12066
[3] Bulashenko O M and Kochelap O V 1991 *Proc. 6th Sci. Conf. on Fluctuation Phenomena in Physical Systems (Palanga, 1991)* ed V Palenskis (Vilnius: Vilnius University Press) p 59
[4] Bulashenko O M, Kochelap O V and Kochelap V A 1992 *Phys. Rev.* B **45** 14308; 1993 *Solid-State Electron.* **36** 111
[5] Kuhn T, Reggiani L and Varani L 1992 *Nuovo Cimento* D **14** 509
[6] Nougier J P, Vaissière J C and Gontrand C 1983 *Phys. Rev. Lett.* **51** 513
[7] Petrosyan S G and Shik A Ya 1989 *Sov. Phys.–JETP* **96** 2229
[8] Pellegrini B 1986 *Phys. Rev.* B **34** 5921
[9] Gantsevich S V, Gurevich V L and Katilius R 1979 *Riv. Nuovo Cimento* **2** N 5
[10] Fuchs K 1938 *Proc. Camb. Phil. Soc.* **34** 100
[11] Beenakker C W J and van Houten H 1988 *Phys. Rev.* B **38** 3232
[12] Bulashenko O M and Kochelap V A 1993 Geometrical resonances in the Nyquist noise spectrum for ballistic conductors *Preprint* ICTP, Trieste
[13] Thornton T J, Roukes M L, Sherer A and Van der Gaag B P 1989 *Phys. Rev. Lett* **63** 2128
Ochiai Y, Abe S, Kawabe M, Ishibashi K, Aoyagi Y, Gamo K and Namba S 1991 *Phys. Rev.* B **43** 14750
Subramaniam S, Bandyopadhyay S, Das B, Bernstein G H and Sekula-Moisé P A 1992 *Phys. Rev.* B **45** 3826